# All pass filters and non-linear optimization for the design of mixed phase inverse filters

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## ABSTRACT

This paper present a new algorithm for mixed phase deconvolution using all pass filter operator designed with a special configuration of roots. All pass filter is used to change the phase of the minimum phase filter. The roots configuration generate a finite wavelet that are implemented in the all-pass filter. The influence of the quantity of roots, around the unit circle and its amplitude, was studied in order to obtain a optimum inverse mixed-phase filter. The optimization of the best filter is evaluated using a genetic algorithm, the maximum value of the varimax norm was evaluated like criterium of simplicity in the reflectivity series. The method was tested using synthetic and real seismic data.

## INTRODUCTION

An important problem in seismic data processing is to improve the temporal resolution, also quality control of phase and amplitude of the wavelet estimated play an important point.

The conventional filters design and wavelet estimation for the deconvolution of seismic data is: the reflectivity is a random process like white noise, the wavelet is stationary and minimum phase, the signal to noise ratio is large, the autocorrelation function (ACF) of the seismic trace is similar to the wavelet (Yilmaz, 1987).

The z-transform minimum phase wavelet has all the roots outside of the unit circle and a optimal filter is design to solving the normal equations with the Levinson algorithm (Robinson and Treitel, 2000), (Claerbout, 1985). If the wavelet is of mixed phase like a the marine seismic wavelet, the filter produces a poor result (Leinbach, 1993), the mixed phase wavelet has roots inside and outside of the unit circle and it only allowed obtain the minimum phase component in the inverse filter.

The solution of the extended Yule Walker equations (ACF with lag $\alpha$ on the main diagonal of the filter equations) is used to estimate roots of the minimum-phase wavelet for generate all pass filters which change the phase of the minimum-phase filter (Porsani and Ursin, 1998), (Ursin and Porsani, 2000).

Higher-order statistics retain phase information, nonminimum phase wavelets is estimated with cepstrum of the fourth-order cumulan, (Sacchi and Ulrych, 2000), (Lu and Wang, 2007), (Velis and Ulrych, 1996). Time-varying Wiener filtering shows the phase changes of the seimic wavelet by constant phase rotation method, (Levi and Oldenburg, 1982), (Bann, 2008).

Deconvolution with mixed phase inverse filter and all-pass operator is very important because extract the information phase of the filter and the wavelet estimation. We propose in this paper a roots configuration method like a alternative work that also have a phase control information.

Several configuration of roots outside of the unit circle was tested to obtain all-pass filter. We study the influence of the number of the roots and its proximity respect to the unity circle, in order to obtain inverse filters. The non linear optimization with a genetic algorithm was implemented for obtains the best filter that make the deconvolution in the seismic data, the output criterion is the simplicity of the result of the deconvolution with the varimax norm maximization (Wiggins, 1978).

## CONVOLUTION MODEL AND MIXED PHASE INVERSE FILTER

The trace $x_t$ is the result of the convolution of the wavelet $p_t$ with the reflectivity series $r_t$ plus the random noise $w_t$.

$$x_t = p_t * r_t + w_t \tag{1}$$

The seismic wavelet with Z-transform is:

$$P(Z) = \sum_{j=0}^{N} p_j Z^j \tag{2}$$

The mixed phase $P(Z)$ has no zeros on the unit circle, $\beta$ zeros inside the unit circle and $N-\beta$ zeros outside the unit circle, also has two component, minimum phase component $A(Z)$ and maximal phase component $Z^{\beta}B(Z^{-1})$ , the pulse is:

$$P(Z) = A(Z)Z^{\beta}B(Z^{-1}) \tag{3}$$

Where the component are:

$$A(Z) = 1 + a_1 Z + \ \dots\ + a_{N-\beta} Z^{N-\beta} \tag{4}$$

$$B(Z^{-1}) = 1 + b_1 Z^{-1} + \ \dots \ + b_\beta Z{-}\beta \quad (5)$$

Now, the seismic wavelet of minimum-phase is:

$$\tilde{P}(Z) = A(Z)B(Z) \quad (6)$$

Together the components, $A(Z)$ and $B(Z)$ are minimum phase, so the combination of the equation (3) and (6) is:

$$P(Z) = \tilde{P}(Z)\frac{Z^\beta B(Z^{-1})}{B(Z)} \quad (7)$$

We show that the mixed-phase wavelet in the equations (7) is the minimum phase wavelet convolved with an all-pass filter. The inverse filter of the mixed phase pulse with respect to the mixed-phase wavelet is:

$$H(Z) = \tilde{H}(Z)\frac{B(Z)}{Z^\beta B(Z^{-1})} \quad (8)$$

Now $B(Z)$ is a artificial finite wavelet designed, the z-transform of the wavelet is like according to the figure 1 observed, the z-transform is also a minimum phase wavelet and has all the roots outside of the unit circle with a ring configuration , the quantity of roots in the rings as also the near or far in the unit radius circle of the z-plane was studied, only certain amount of roots and radius allowed a good design of the all-pass operator, after it was implemented in the all pass filter of the equation (8).

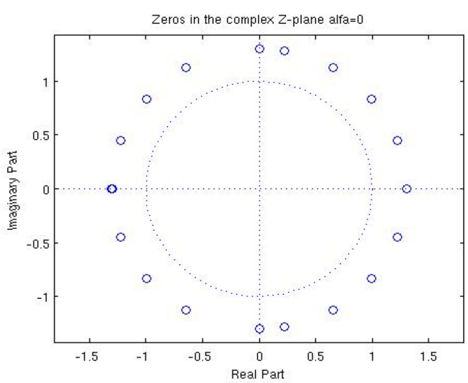


Figure 1: the roots outside of the unit circle with a ring configuration.

## DECONVOLUTION WITH MIXED-PHASE INVERSE FILTER

When the wavelet is not known the autocorrelation function is estimated directly of the seismic data, where the reflectivity series and noise are white, only it is designed to solving the normal equations with the Levinson algorithm and obtain the minimum phase component in the inverse filter. The minimum phase component filter is implemented in the design of the filter of the equation (8). The inverse mixed phase filter computed is applied on the seismic trace, the deconvolved trace is:

$$e_t = x_t * h_t = r_t * p_t * h_t + w_t$$

$p_t * h_t = \delta_t$ where $\delta_t$ is the delta de Kronecker. If the signal/nose ratio large we have $e_t \approx r_t$.

The deconvolution is much approximated with the reflectivity series.

## OPTIMUM MIXED PHASE INVERSE FILTER AND NUMERICAL RESULTS

The figure 2 shows the application of a mixed phase inverse filter designed with the all-pass filter, artificial finite wavelet with 20 roots was implemented, see in the figure 2: a) is the synthetic mixed phase wavelet, b) is the mixed phase filter performance very good in the mixed phase wavelet, c) shows the deconvolution of the mixed phase filter and the mixed phase wavelet, the result is a perfect spike and d) is the minimum phase filter, where has all the roots outside of the unit circle, the inverse filter is causal and minimum phase, but to produces a poor result because it has only a causal component.

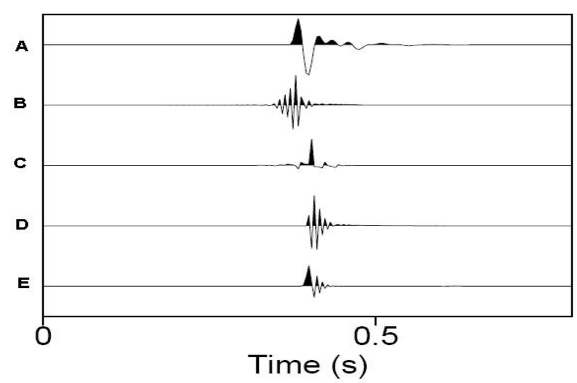


Figure 2: Deconvolution of a single mixed phase wavelet with the mixed phase inverse filter.

the figure 3 shows the performance of the genetic algorithm with the varimax norm, it was applied in the synthetic data with a single wavelet, see figure 2a, the three lines shows the maximum, minimum and mean value of the varimax norm, observing the convergence of the algorithm, directing the maximum value.

The figure 4 show the results using a genetic algorithm (GA) to evaluate an optimal mixed phase inverse filter, it is designed with a all-pass filter, in the all-pass filter also was implemented a artificial finite wavelet with ring configuration in the z-plane. The figure 4 is : a) the random reflectivity series, b) shows the convolution between the mixed phase wavelet and the random reflectivity series, c) is the optimal mixed phase inverse filter and d) is the result of the deconvolution with the optimal mixed phase inverse filter.

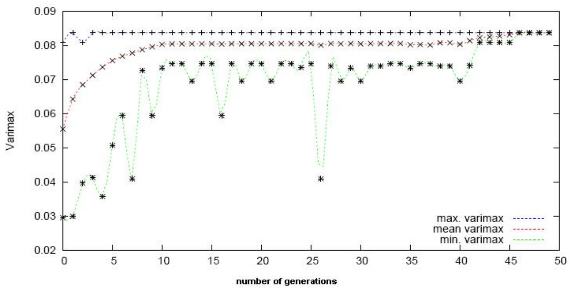


Figure 3: The performance of the genetic algorithm.

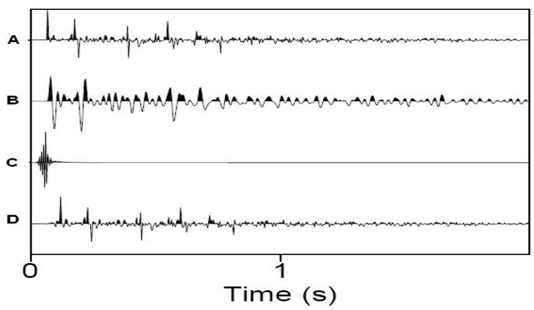


Figure 4: Deconvolution with the optimal mixed phase inverse filter in a synthetic trace.

The figure 5 shows the result of the deconvolution with the minimum phase Wiener-Levinson filter in a synthetic trace, a poor result because it has only a causal component.

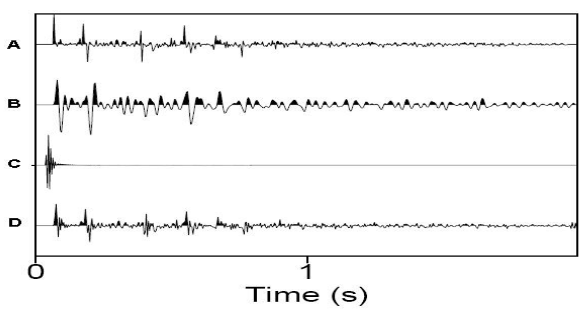


Figure 5: Deconvolution with the minimun phase Winner-Levinson filter in a synthetic trace.

**OPTIMUM MIXED PHASE INVERSE FILTER ON REAL SEISMIC DATA**

The figure 6 shows a real 3D seismic data, is a block in the Dutch sector of the north sea: inline range 100-750, crossline Range 300-1250 and z range of 0-1848. Was seleted a seismic section, a crossline 321, in the figure 7 for a single trace in the section was estimated: a) the minimum phase wavelet, b) the mixed phase wavelet and c) the mixed phase inversed filter, and d) the pulse deconvolved with the mixed phase inverse filter with a good spike.

The figure 8 shows: a) the amplitude spectrum of the seismic section (figure 10A), b) the amplitude spectrum of the deconvolution with the Wiener-Levinson filter (figure 10B) and c) the amplitude spectrum of the deconvolution with the optimal mixed phase inverse filter (figure 10C), this result shows that the amplitude spectrum is similar with the Wiener Levinson deconvolution and the optimal mixed phase inverse filter because the all pass filter only change the phase, equation (8).

The figure 9 shows: a) the real trace selected of the seismic section, b) the result of the deconvolution with the Wiener-Levinson filter and c) the result of the deconvolution with the optimal mixed phase inverse filter.

The figure 10 shows: A) the seismic section selected in the 3D seismic data (crossline 321), B) the Wiener-Levinson deconvolution and C) the optimal mixed phase inverse filter with to improve temporal resolution.

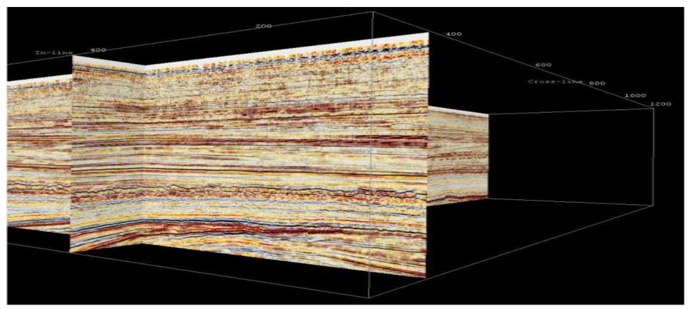

Figure 6: 3D seismic data, Netherlands Offshore F3 Block, Opendect.

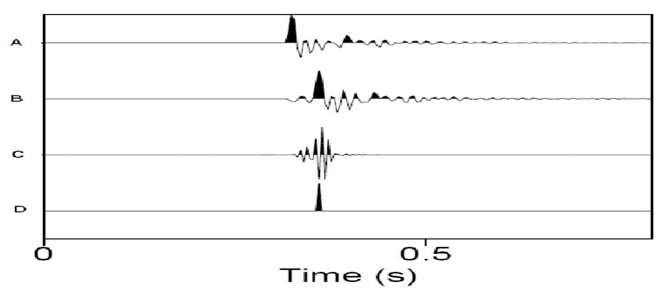


Figure 7: The wavelet estimated and the deconvolution with the mixed phase inversed filter(perfect spike).

**CONCLUSIONS**

An algorithm with mixed phase inverse filter for deconvolution was study and applied in real and synthetic seismic data. The filters were generated with different all-pass filter, it was designed with several configuration of roots. A genetic algorithm (GA) was evaluated to obtain a optimal mixed phase inverse filter, with good results in the deconvolution in a 3D seismic cube.

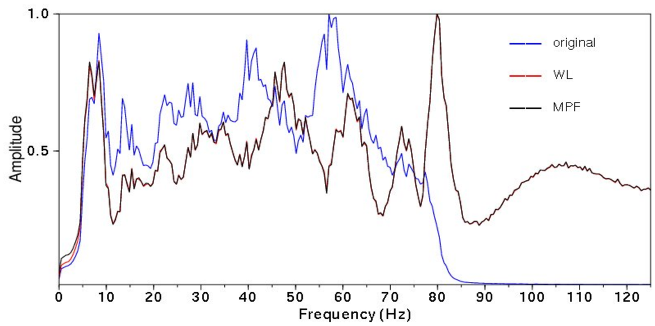


Figure 8: Original: amplitude spectrum of the real siesmic trace, WL: amplitude spectrum with WL deconvolution, MPF: amplitude spectrum with the mixed phase inversed filter deconvolution.

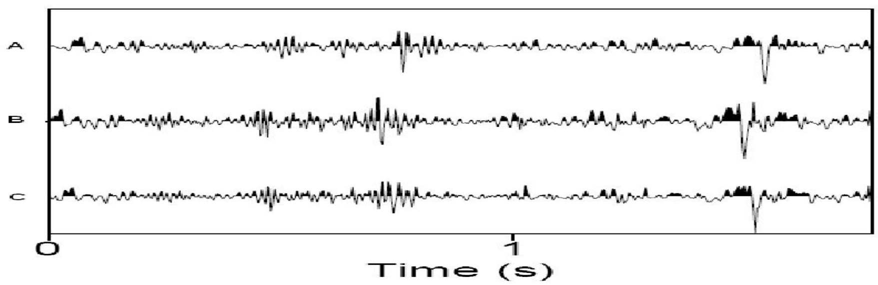


Figure 9: a) the real siesmic trace, b) deconvolution with Wiener Levinson filter, c) deconvolution with the mixed phase inversed filter.

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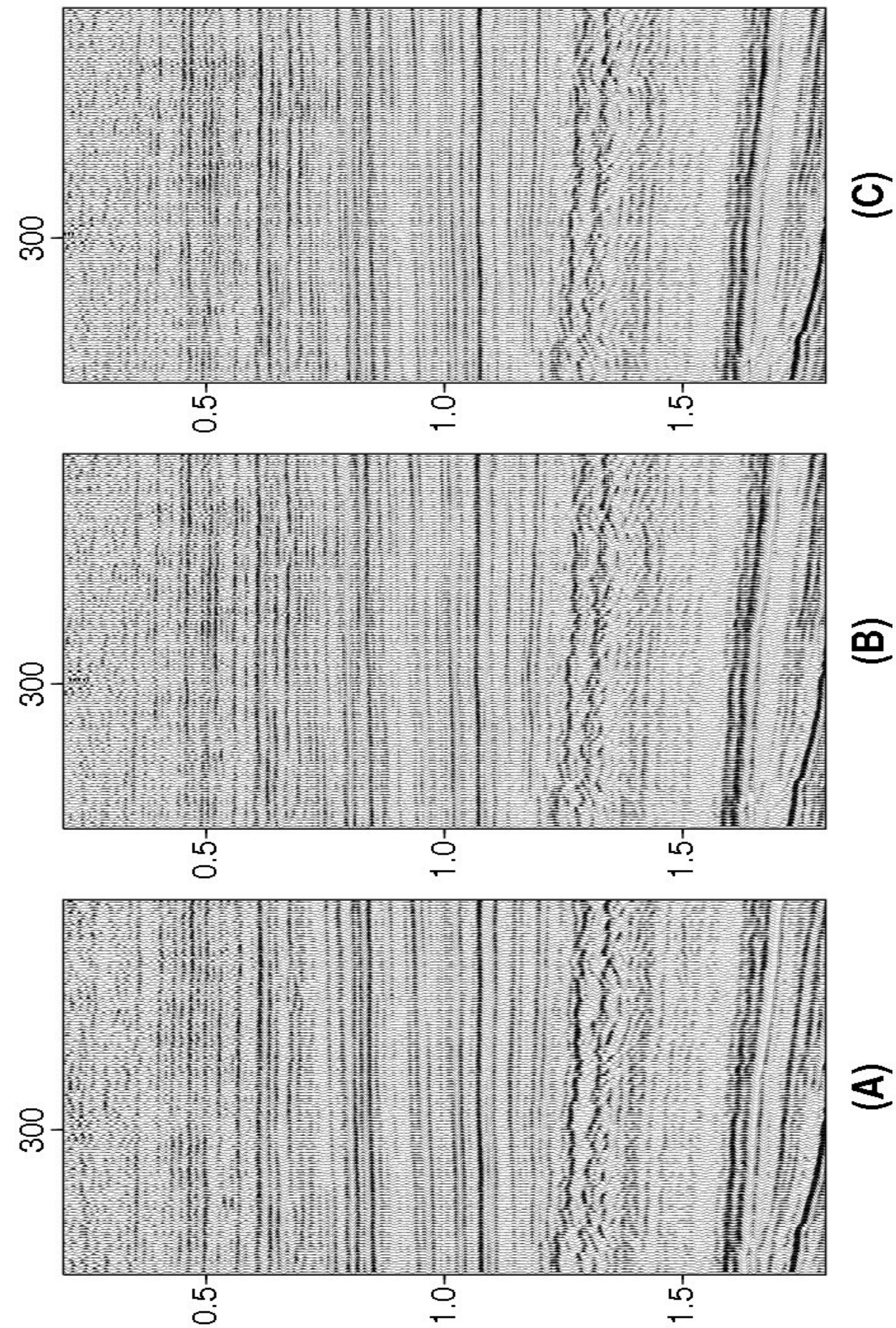


Figure 10: the figure shows : A) the seismic section selected in the 3D seismic data (crossline 321), B) the Wiener-Levinson deconvolution and C) the deconvolution with mixed phase inverse filter.